Root Approximation

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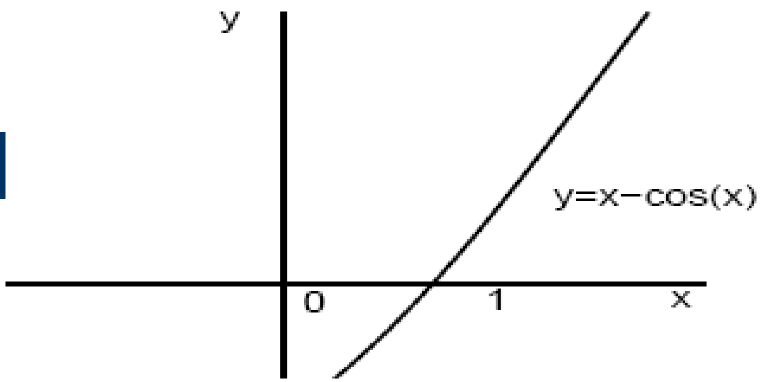
Bisection method



$f < 0$ at $x =$	$f > 0$ at $x =$	mid point	f	approximate answer	Error Bound
0.0	1.5	0.75	+ve	0.75	$\pm$ 0.75

$\therefore$ root lies between 0.0 and 0.75.

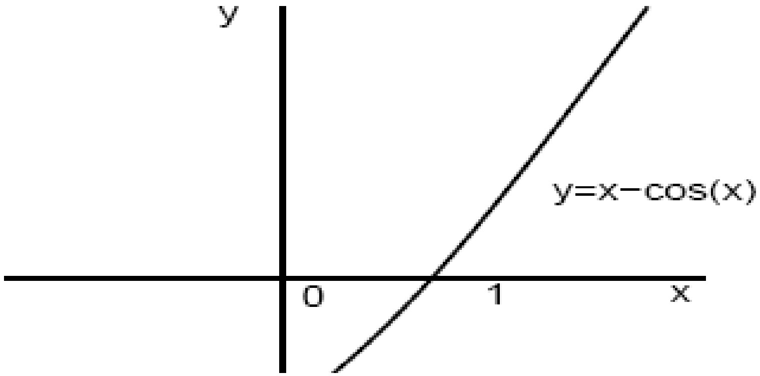
0.0	0.75	0.375	-ve	0.375	$\pm$ 0.375

$\therefore$ root lies between 0.375 and 0.75.

0.375	0.75	0.5625	-ve	0.5625	$\pm$ 0.1875

$\therefore$ root lies between 0.5625 and 0.75.

Bisection method



$f < 0$ at $x =$	$f > 0$ at $x =$	mid point	f (mid point)
0.0	1.5	0.75	+
0.0	0.75	0.375	−
0.375	0.75	0.5625	−
0.5625	0.75	0.65625	−
0.65625	0.75	0.70313	−
0.70313	0.75	0.72656	−
0.72656	0.75	0.73828	−
0.73828	0.75	0.74414	+
0.73728	0.74414	0.74121	+

Algorithm

```
// x1:left point   //x2:right point
fx = 0.5*(x1+x2);  // fx :mid-point
y = fx - cos(fx);
if(y<0)x1=fx;
if(y>0)x2=fx;
if(fabs(y)<1e-10)
{
     root=fx;
     cout<<"root at "<<root<<endl;
     break;
}
```

Ex1.

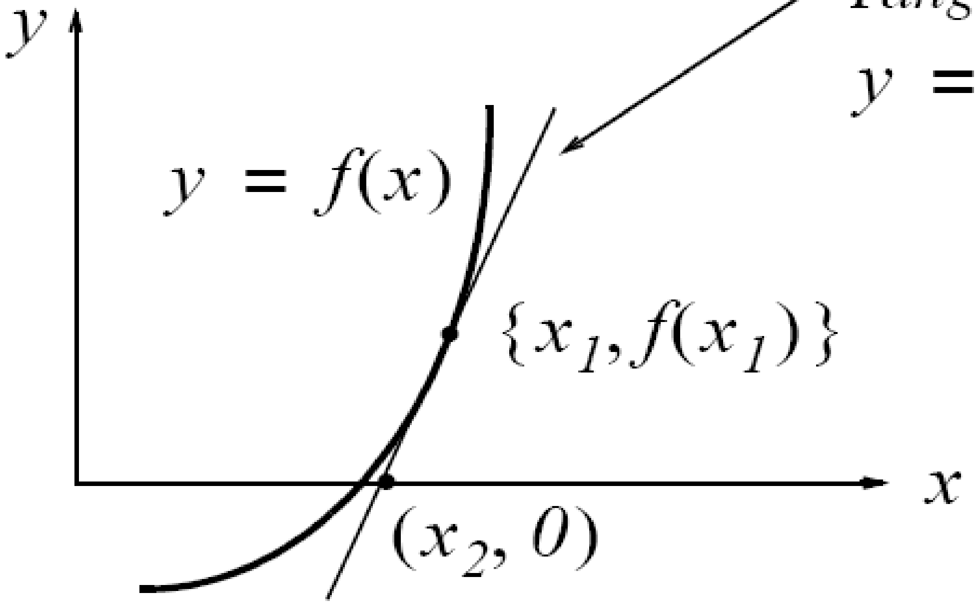
Use the bisection method to find the root of :

$$y = x - \cos(x)$$

Newton-Raphson method

- *Newton's* (*or Newton-Raphson*) method is an iterative (repetitive procedure) method that can be used to approximate the roots of any linear or nonlinearequation of any degree.
- Assume that the slope is neither zero nor infinite. Then, the slope (first derivative) at *x=x1 is*

Newton-Raphson method



Tangent line (slope) to the curve $y = f(x)$ *at point* $\{x_1, f(x_1)\}$

$$f'(x_1) = \frac{y - f(x_1)}{x - x_1}$$

Newton-Raphson method

$$y - f(x_1) = f'(x_1)(x - x_1)$$

The slope crosses the X-axis at x=x2 and y=0. Since this point [*x2*, *f*(*x2*)] = (*x2*, *0*) lies on the slope line, By substitution,

$$0 - f(x_1) = f'(x_1)(x_2 - x_1)$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Algorithm

```
//  f(x) = x^2-x-1
fx = x*x - x -1.0;
//  f'(x) = 2x
df = 2*x - 1.0;
dx = fx/df;
x = x - dx;
if (fabs(dx) < 1e-10)root=x;
```

Ex.2

Approximate root of the polynomial equation:

$$y = f(x) = x^3 - 7x^2 + 16x - 12$$

Hint :

```
fx = pow(x,3) - 7*pow(x,2) + 16*x - 12;
df = 3*pow(x,2) -7*x + 16;
```

Ex.3

Approximate one real root of the non-linear equation

$$f(x) = x^2 + 4x + 3 + \sin x - x\cos x$$

Try at least three initial values.

Hint : x = -0.89….